

# Double Beta Decay in Gauge Theories

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**Abstract.** Neutrinoless double beta decay is a very important process both from the particle and nuclear physics point of view. From the elementary particle point of view it pops up in almost every model. In addition to the traditional mechanisms, like the light neutrino mass,  $\lambda$  and  $\eta$  terms etc we can have direct R-parity violating supersymmetric (SUSY) contributions. In any case its observation will severely constrain the existing models and will signal that the neutrinos are massive Majorana particles. From the nuclear physics point of view it is challenging, because: 1) The relevant nuclei have complicated nuclear structure. 2) The energetically allowed transitions are exhaust a small part of all the strength. 3) One must cope with the short distance behavior of the transition operators, especially when the intermediate particles are heavy (eg in SUSY models). Thus novel effects, like the double beta decay of pions in flight between nucleons, have to be considered. 4) The intermediate momenta involved are about  $100 \text{ MeV}/c$ . Thus one has to take into account possible momentum dependent terms in the nucleon current. We find that, for the mass mechanism, such modifications of the nucleon current for light neutrinos reduce the nuclear matrix elements by about 25%, almost regardless of the nuclear model. In the case of heavy neutrino the effect is much larger and model dependent. Taking the above effects into account, the available nuclear matrix elements for the experimentally interesting nuclei  $A = 76, 82, 96, 100, 116, 128, 130, 136$  and  $150$  and the presently available experimental limits on the half-life of the  $0\nu\beta\beta$ -decay we have extracted the following

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new limits:  $\langle m_\nu \rangle < 0.3 eV/c^2$   $\lambda'_{111} < 4.0 \times 10^{-4}$  for the R-parity violating parameter with reasonable choices of the parameters of SUSY models,

## 1. Introduction

The nuclear double beta decay can occur whenever the ordinary (single) beta decay is forbidden due to energy conservation or greatly suppressed due to angular momentum mismatch. The exotic neutrinoless double beta decay ( $0\nu\beta\beta$ -decay) is the most interesting since it violates lepton number by two units. It is a very old process. It was first considered by Furry [1] exactly half a century ago as soon it was realized that the neutrino might be a Majorana particle. It was continued with the work of Primakoff and Rosen [2] especially when it was recognized that kinematically it is favored by  $10^8$  compared to its non exotic sister  $2\nu\beta\beta$ -decay. When the corresponding level of the  $10^{15}y$  lifetime was reached and the process was not seen, it was tempting to interpret this as an indication that the neutrino was a Dirac particle. The interest in it was resurrected with the advent of gauge theories which favor Majorana neutrinos and through the pioneering work of Kotani and his group [3] it was brought again to the attention of the nuclear physics community. To-day, fifty years later,  $0\nu\beta\beta$ -decay continues to be one of the most interesting processes.

From a theoretical point of view it is the most likely, if not the only, process capable of deciding whether or not the neutrino is a Majorana particle, i.e. it coincides with its own antiparticle [4, 5, 6, 7, 8, 9]. It is expected to occur whenever one has lepton number violating interactions. Lepton number, being a global quantity is not sacred, but it is expected to be broken at some level. In short this process pops up almost everywhere, in every theory.

From a nuclear physics point of view calculating the relevant nuclear matrix elements it is indeed a challenge. First almost all nuclei, which can undergo double beta decay, are far from closed shells and some of them are even deformed. One thus faces a formidable task. Second the nuclear matrix elements represent a small fraction of a canonical value ( i.e. the matrix element to the energy non allowed transition double Gamow-Teller resonance or some appropriate sum rule). Thus effects which are normally negligible become important here. Third in many models the dominant mechanism for  $0\nu\beta\beta$ -decay does not involve intermediate light neutrinos, but very heavy particles and one must be able to cope with the short distance behavior of the relevant operators and wave functions.

From the experimental point of view it is also very challenging to measure the slowest perhaps process accessible to observation. Especially since it is realized that, even if one obtains only lower bounds on the life time for this decay, the extracted limits on the theoretical model parameters may be comparable, if not better, and complementary to those extracted from the most ambitious accelerator experiments.

The recent superkamiokande results have given the first evidence of physics beyond the Standard Model (SM) and in particular they indicate that the neutrinos are massive particles. It is important to proceed further and find out whether the neutrinos are Dirac or Majorana particles. As we have mentioned there might be processes other than the conventional intermediate neutrino mechanism, which may dominate  $0\nu\beta\beta$ -decay. It has, however, been known that whatever the lepton violating process is, which gives rise to this decay, it can be used to generate a Majorana mass for the neutrino [10]. The study of the  $0\nu\beta\beta$ -decay is further stimulated by the development of grand unified theories (GUT's) and supersymmetric models (SUSY) representing extensions of the  $SU(2)_L \otimes U(1)$  SM. The GUT's and SUSY offer a variety of mechanisms which allow the  $0\nu\beta\beta$ -decay to occur [11].

The best known possibility is via the exchange of a Majorana neutrino between the two decaying neutrons [4, 5, 6, 7, 12, 9, 8]. Nuclear physics dictates that we study the light and heavy neutrino components separately. In the presence of only left-handed currents for light intermediate neutrinos the obtained amplitude is proportional to a suitable average neutrino mass, which vanishes in the limit in which the neutrinos become Dirac particles. In the case of heavy Majorana neutrino components the amplitude is proportional to the average inverse neutrino mass, i.e. it is again suppressed. In the presence of right handed currents one has one can have a contribution similar to the one above for heavy neutrinos but involving a different (larger) average inverse mass and some suppression due to the heaviness of  $W_R$ .

It is also possible to have, in addition, interference between the leptonic left and right currents,  $j_L - j_R$  interference. In this case the amplitude in momentum space becomes proportional to the 4-momentum of the neutrino and, as a result, only the light neutrino components become important. One now has two possibilities. First the two hadronic currents have a chirality structure of the same kind  $J_L - J_R$ . Then one can extract from the data a dimensionless parameter  $\lambda$ , which is proportional to the square of the ratio of the masses of the L and R gauge bosons,  $\kappa = (m_L/m_R)^2$ . Second the two hadronic currents are left-handed, which can happen via the mixing of the two bosons. The relevant lepton violating parameter  $\eta$  is now proportional to this mixing. Both of these parameters, however, involve the neutrino mixing and they are proportional to the mixing between the light

and heavy neutrinos.

In gauge theories one has, of course, many more possibilities. Exotic intermediate scalars may mediate  $0\nu\beta\beta$ -decay [6]. These are not favored in current gauge theories and are not going to be further discussed. In superstring inspired models one may have siglet fermions in addition to the usual right handed neutrinos. Not much progress has been made on the phenomenological side of these models and they are not going to be discussed further.

In recent years supersymmetric models are taken seriously and semirealistic calculations are taking place. In standard calculations one invokes universality at the GUT scale, employing in all 5 parameters, and use the renormalization group equation to obtain all parameters (couplings and particle masses) at low energies. Hence, since such parameters are in principle calculable one can use  $0\nu\beta\beta$ -decay to constrain some of the R-parity violating couplings, which cannot be specified by the theory [13, 14, 15, 16, 17, 18, 19]. Recent review articles [9, 8] give a detailed account of the latest developments in this field.

From the above discussion it clear that one has to consider the case of heavy intermediate particles. One thus has to consider very short ranged operators in the presence of the nuclear repulsive core. If the interacting nucleons are point-like one gets negligible contributions. We know, however that the nucleons are not point like and they have structure described by a form factor, which can be calculated in the quark model or parameterized in a dipole shape. This approach was first considered by Vergados [20] adopted later by almost everybody. The resulting effective operator has a range somewhat less than the proton mass (see sect. 4 below).

The other approach is to consider particles other than the nucleons present in the nuclear soup. For  $0^+ \rightarrow 0^+$  the most important such particles are the pions. One thus may consider the double beta decay of pions in flight between nucleons, like

$$\pi^- \longrightarrow \pi^+ e^- e^- \quad , \quad n \longrightarrow p \pi^+ e^- e^- \quad (1)$$

This contribution was first considered by Vergados [21] and was found to yield results of the same order as the nucleon mode with the above recipe for treating the short range behavior. It was revived by the Tuebingen group [17, 18] in the context of R-parity violating interactions, in which it appears to dominate.

The other recent development is the better description of nucleon current by including momentum dependent terms, such as the modification of the axial current due to PCAC and the inclusion of the weak magnetism terms. These contributions have been considered previously [22, 12], but only in connection with the extraction of the  $\eta$  parameter mentioned above. Indeed these terms were very important in this case since they compete with

the p-wave lepton wave function, which, with the usual currents, provides the lowest non vanishing contribution. In the mass term, however, only s-lepton wave functions are relevant. So these terms have hitherto been neglected.

It was recently found [23] that for light neutrinos the inclusion of these momentum dependent terms reduces the nuclear matrix element by about 25%, independently of the nuclear model employed. For the heavy neutrino, however, the effect can be larger and depends on the nuclear wave functions. The reason for expecting them to be relevant is that the average momentum  $\langle q \rangle$  of the exchanged neutrino is expected to be large [24]. In the case of a light intermediate neutrino the mean nucleon-nucleon separation is about 2 fm which implies that the average momentum  $\langle q \rangle$  is about 100 MeV. In the case of a heavy neutrino exchange the mean internucleon distance is considerably smaller and the average momentum  $\langle q \rangle$  is supposed to be considerably larger.

Since  $0\nu - \beta\beta$  decay is a two step process, one should in principle construct and sum over all the intermediate nuclear steps, a formidable job indeed in the case of the Shell Model Calculations (SMC). Since, however, the average neutrino momentum is much larger compared to the nuclear excitations, one can invoke closure using some average excitation energy (this does not apply in the case of  $2\nu\beta\beta$  decays). Thus one need construct only the initial and final  $0^+$  nuclear states. In Quasiparticle Random Phase Approximation (QRPA) one must construct the intermediate states anyway. In any case it was explicitly shown, taking advantage of the momentum space formalism developed by Vergados [25], that this approximation is very good [26, 27]. The same conclusion was reached independently by others [28].

Granted that one takes into account all the above ingredients in order to obtain quantitative answers for the lepton number violating parameters from the the results of  $0\nu\beta\beta$ -decay experiments, it is necessary to evaluate the relevant nuclear matrix elements with high reliability. The most extensively use methods are the SMC ( for a recent review see [8]) and QRPA( for a recent review see [9, 8]). The SMC is forced to use few single particle orbitals, while this restriction does not apply in the case of QRPA. The latter suffers , of course, from the approximations inherent in the RPA method. So a direct comparison between them is not possible.

The SMC has a long history [29, 30, 31, 32, 33, 34, 35] in in double beta decay calculations. In recent years it has lead to large matrices calculations in traditional as well as Monte Carlo types of calculations [36, 37, 38, 39, 40, 41] (For a more complete set of references see Ref. [8]) and suitable effective interactions.

There have been a number of QRPA calculations covering almost all nuclear targets [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52]. We also have

seen some refinements of QRPA, like proton neutron pairing and inclusion of renormalization effects due to Pauli principle corrections [53, 54].

The above schemes, in conjunction with the other improvements mentioned above offer, some optimism in our efforts for obtaining nuclear matrix elements accurate enough to allow us to extract reliable values of the lepton violating parameters from the data. We will review this in the case of most of the nuclear targets of experimental interest ( $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{96}\text{Zr}$ ,  $^{100}\text{Mo}$ ,  $^{116}\text{Cd}$ ,  $^{128}\text{Te}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ,  $^{150}\text{Nd}$ ).

## 2. Theory

### 2.1. Majorana neutrino mass mechanism

We shall consider the  $0\nu\beta\beta$ -decay process assuming that the effective beta decay Hamiltonian acquires the form:

$$\mathcal{H}^\beta = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1 - \gamma_5)\nu_{eL}^0] J_L^{\mu\dagger} + [\bar{e}\gamma_\mu(1 + \gamma_5)\nu_{eR}^0] J_R^{\mu\dagger} + h.c. \quad (2)$$

where  $e$  and  $\nu_{eL}^0$ ,  $\nu_{eR}^0$  are field operators representing the electron and the left handed and the right handed electron neutrinos in the weak interaction basis, respectively. We suppose that neutrino mixing does take place according to

$$\nu_{eL}^0 = \sum_{k=1}^3 U_{ek}^{(11)} \nu_{kL} + \sum_{k=1}^3 U_{ek}^{(12)} N_{kL}, \quad (3)$$

$$\nu_{eR}^0 = \sum_{k=1}^3 U_{ek}^{(21)} \nu_{kL} + \sum_{k=1}^3 U_{ek}^{(22)} N_{kL}, \quad (4)$$

where,  $\nu_k$  ( $N_k$ ) are fields of light (heavy) Majorana neutrinos with masses  $m_k$  ( $m_k \ll 1$  MeV) and  $M_k$  ( $M_k \gg 1$  GeV), respectively. The matrices  $U_{ek}^{(11)}$  and  $U_{ek}^{(22)}$  are approximately unitary, while the matrices  $U_{ek}^{(12)}$  and  $U_{ek}^{(21)}$  are very small (of order of the up quark divided by the heavy neutrino mass scales) so that the overall matrix is unitary.  $\nu_k, N_k$  satisfy the Majorana condition:  $\nu_k \xi_k = C \bar{\nu}_k^T$ ,  $N_k \Xi_k = C \bar{N}_k^T$ , where  $C$  denotes the charge conjugation and  $\xi, \Xi$  are phase factors (the eigenmasses are assumed positive).

We assume both outgoing electrons to be in the  $s_{1/2}$  state and consider only  $0_i^+ \rightarrow 0_f^+$  transitions are allowed. For the ground state transition restricting ourselves to the mass mechanism we obtain for the  $0\nu\beta\beta$ -decay inverse half-life [4, 5, 6, 7, 12, 9, 8],

$$[T_{1/2}^{0\nu}]^{-1} = G_{01} \left[ \frac{\langle m_\nu \rangle}{m_e} M_{<m_\nu>}^{light} + |\eta_N^L M_{\eta_N}^{heavy}|^2 + |\eta_N^R M_{\eta_N}^{heavy}|^2 \right] \quad (5)$$

The lepton-number non-conserving parameters, i.e. the effective neutrino mass  $\langle m_\nu \rangle$  and  $\eta_N^L, \eta_N^R$  are given as follows:

$$\langle m_\nu \rangle = \sum_1^3 (U_{ek}^{(11)})^2 \xi_k m_k, \quad \eta_N^L = \sum_1^3 (U_{ek}^{(12)})^2 \Xi_k \frac{m_p}{M_k}, \quad (6)$$

$$\eta_N^R = (\kappa^2 + \epsilon^2) \sum_1^3 (U_{ek}^{(22)})^2 \Xi_k \frac{m_p}{M_k}, \quad (7)$$

with  $m_p$  ( $m_e$ ) being the proton (electron) mass,  $\kappa$  is the mass squared ratio of  $W_L$  and  $W_R$  and  $\epsilon$  their mixing.  $G_{01}$  is the integrated kinematical factor [5, 12]. The nuclear matrix elements associated with the exchange of light ( $M_{\langle m_\nu \rangle}^{light}$ ) and heavy neutrino ( $M_{\eta_N}^{heavy}$ ) must be computed in a nuclear model. Eq. (5), however, applies to any intermediate particle.

At this point we should stress that the main suppression in the mass terms comes from the smallness of neutrino masses. In the case of heavy neutrino not only from the large values of neutrino masses but the small couplings,  $U^{(12)}$  for the left handed neutrinos and  $\kappa$  and  $\epsilon$  for the right-handed ones.

## 2.2. The leptonic left-right interference mechanism ( $\lambda$ and $\eta$ terms).

As we have already mentioned in the presence of right handed currents one can have interference between the leptonic currents of opposite chirality. This leads to different kinematical functions and two new lepton violating parameters  $\lambda$  and  $\eta$  defined by

$$\eta = \epsilon \eta_{RL} \quad , \quad \lambda = \kappa \eta_{RL} \quad , \quad \eta_{RL} = \sum_1^3 (U_{ek}^{(21)} U_{ek}^{(11)}) \xi_k \quad (8)$$

The parameters  $\lambda$  and  $\eta$  are small not only due to the smallness of the parameters  $\kappa$  and  $\epsilon$  but in addition because of the smallness of  $U_{(21)}$ .

All the above contributions vanish in the limit in which the neutrino is a Dirac particle.

Many nuclear matrix elements appear in this case, but they are fairly well known and they are not going to be reviewed here (see e.g. [4, 6] and [5, 7, 8, 9] and in our notation [12]). We only mention that in the case of the  $\eta$  we have additional contributions coming from the nucleon recoil term and the kinematically favored spin antisymmetric term. These dominate and lead to values of  $\eta$  much smaller than  $\lambda$  [12].

### 3. The R-parity violating contribution

In SUSY theories R-parity is defined as

$$R = (-1)^{3B+L+2s} \quad (9)$$

with  $B = baryon$ ,  $L = lepton$  numbers and  $s$  the spin. It is +1 for ordinary particles and -1 for their superpartners. R-parity violation has recently been seriously considered in SUSY models.

R-parity violating terms may induce a Majorana neutrino mass and may, therefore, lead to  $0\nu\beta\beta$  decay. But the relevant masses are small, at least 10 times smaller [55] than those deduced from the present data. The bilinear terms in the superpotential also lead to mixings between the neutrinos and neutralinos as well as between the leptons and the charginos, leading to lepton violating processes [56]. We are not going, however, to consider such effects in this review.

Here we will be concerned with trilinear couplings in the superpotential given by:

$$W = \lambda_{ijk} L_i^a L_j^b E_k^c \epsilon_{ab} + \lambda'_{ijk} L_i^a U_j^b D_k^c \epsilon_{ab} + \lambda''_{ijk} U_i^c U_j^c D_k^c \quad (10)$$

where a summation over the flavor indices  $i,j,k$  and the isospin indices  $a,b$  is understood (  $\lambda_{ijk}$  is antisymmetric in the indices  $i$  and  $j$ ) The last term has no bearing in our discussion, but we will assume that it vanishes due to some discrete symmetry to avoid too fast proton decay. The  $\lambda$ 's are dimensionless couplings not predicted by the theory.

In the above notation  $L,Q$  are isodoublet and  $E^c, D^c$  isosinglet chiral superfields, i.e they represent both the fermion and the scalar components. It has been recognized quite sometime ago that the second term in the superpotential could lead to neutrinoless double beta decay [13, 14] and re-examined quite recently [17]. Typical diagrams at the quark level are shown in Fig.1. Note that as intermediate states, in addition to the s-leptons and s-quarks, one must consider the neutralinos, 4 states which are linear combinations of the gauginos and higgsinos, and the colored gluinos (supersymmetric partners of the gluons). Whenever the process is mediated by gluons a Fierz transformation is needed to lead to a colorless combination. The same thing is necessary whenever the fermion line connects a quark to a lepton. As a result one gets at the quark level not only scalar (S) and pseudoscalar (P) couplings, but tensor (T) couplings as well. This must be contrasted to the V and A structure of the traditional mechanisms. One, therefore, must consider how to transform these operators from the quark to the nucleon level.

The effective lepton violating parameter, assuming that pion exchange



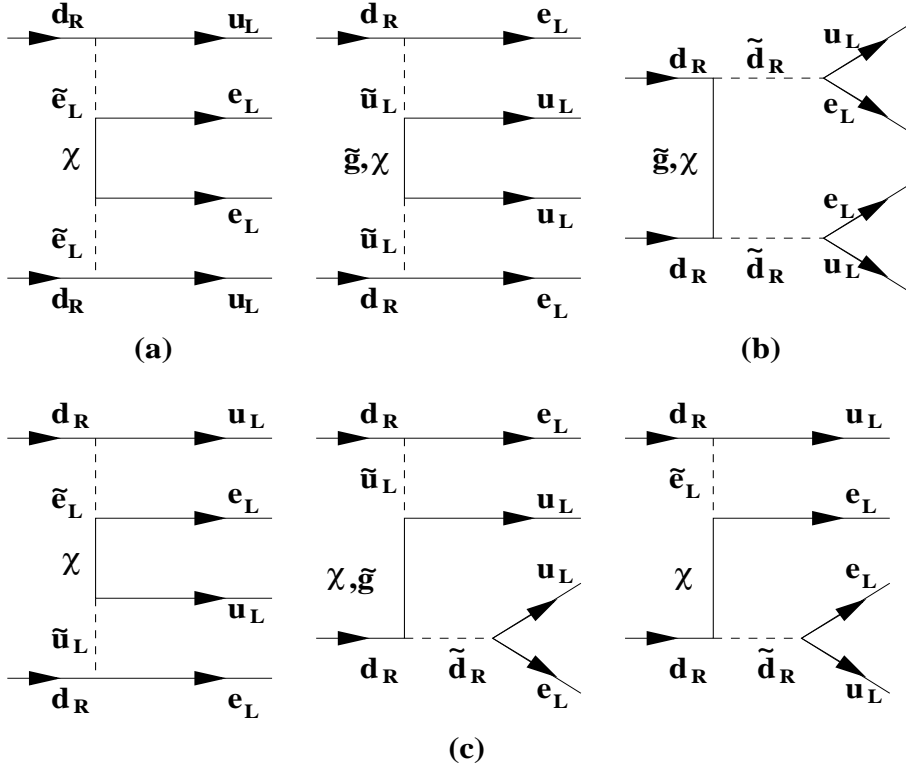


Figure 1. The R-parity violating contribution to  $0\nu\beta\beta$  decay mediated by s-fermions and neutralinos (gluinos).

mode dominates, as the authors of Ref [17, 9] claim, is given by

$$\eta_{SUSY} = (\lambda'_{111})^2 \frac{3}{8} (\chi_{PS} \eta_{PS} + \eta_T) \quad (11)$$

with  $\eta_{PS}(\eta_T)$  associated with the scalar and pseudoscalar (tensor) quark couplings given by

$$\eta_{PS} = \eta_{\tilde{\chi},\tilde{e}} + \eta_{\tilde{\chi},\tilde{q}} + \eta_{\tilde{\chi},\tilde{f}} + \tilde{\eta}_{\tilde{g}} + 7\eta'_{\tilde{g}} \quad , \quad \eta_T = \eta_{\tilde{\chi},\tilde{q}} - \eta_{\tilde{\chi},\tilde{f}} + \tilde{\eta}_{\tilde{g}} - \eta'_{\tilde{g}} \quad (12)$$

They find  $\chi_{PS} = (2/3)$ , but as we shall see it depends on ratios of nuclear matrix elements. For the diagram of Fig.1a one finds

$$\eta_{\tilde{\chi},\tilde{e}} = \frac{2\pi\alpha}{(G_F m_W^2)^2} (\kappa_{\tilde{e}})^2 \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{e}\tilde{e}} \quad (13)$$

For the diagram of Fig.1b one finds

$$\tilde{\eta}_{\tilde{\chi},\tilde{q}} = \frac{\pi\alpha}{2(G_F m_W^2)^2} [(\kappa_{\tilde{d}})^2 \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{d}\tilde{d}} + (\kappa_{\tilde{u}})^2 \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{u}\tilde{u}}] \quad (14)$$

$$\tilde{\eta}_{\tilde{g}} = \frac{\pi}{6} \alpha_s \frac{1}{(G_F m_W^2)^2} [(\kappa_{\tilde{d}})^2 + (\kappa_{\tilde{u}})^2] \frac{m_p}{m_{\tilde{g}}} \quad (15)$$

For the diagram of Fig.1c one finds

$$\tilde{\eta}_{\tilde{\chi},\tilde{f}} = \frac{\pi\alpha}{2(G_F m_W^2)^2} [\kappa_{\tilde{e}} \kappa_{\tilde{d}} \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{e}\tilde{d}} + \kappa_{\tilde{e}} \kappa_{\tilde{u}} \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{e}\tilde{u}} + \kappa_{\tilde{d}} \kappa_{\tilde{u}} \langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{d}\tilde{u}}] \quad (16)$$

$$\tilde{\eta}_{\tilde{g}'} = \frac{\pi}{12} \alpha_s \frac{1}{(G_F m_W^2)^2} \kappa_{\tilde{d}} \kappa_{\tilde{u}} \frac{m_p}{m_{\tilde{g}'}} \quad (17)$$

where

$$\kappa_X = \left(\frac{m_W}{m_X}\right)^2, \quad X = \tilde{e}_L, \tilde{u}_L, \quad \kappa_{\tilde{d}} = \left(\frac{m_W}{m_{\tilde{d}_R}}\right)^2 \quad (18)$$

$$\langle \frac{m_p}{m_{\tilde{\chi}}} \rangle_{\tilde{f}\tilde{f}'} = \sum_1^4 \epsilon_{\tilde{\chi}_i,\tilde{f}} \epsilon_{\tilde{\chi}_i,\tilde{f}'} \frac{m_p}{m_{\tilde{\chi}_i}} \quad (19)$$

where  $\epsilon_{\tilde{\chi}_i,\tilde{f}}$  and  $\epsilon_{\tilde{\chi}_i,\tilde{f}'}$  are the couplings of the  $i^{th}$  neutralino to the relevant fermion-sfermion, which are calculable ( see e.g Ref. [57]). Thus ignoring the small Yukawa couplings coming via the Higgsinos and taking into account only the gauge couplings we find

$$\epsilon_{\tilde{\chi}_i,\tilde{e}} = \frac{Z_{2i} + \tan\theta_W Z_{1i}}{\sin\theta_W} \quad (20)$$

$$\epsilon_{\tilde{\chi}_i,\tilde{u}} = \frac{Z_{2i} + (\tan\theta_W/3)Z_{1i}}{\sin\theta_W}, \quad \epsilon_{\tilde{\chi}_i,\tilde{d}} = -\frac{Z_{1i}}{3\cos\theta_W} \quad (21)$$

where  $Z_{1i}, Z_{2i}$  are the coefficients in the expansion of the  $\tilde{B}, \tilde{W}_3$  in terms of the neutralino mass eigenstates. Note that in this convention some of the masses  $m_{\tilde{\chi}_i}$  may be negative.

#### 4. The effective nucleon current

As we have mentioned the effective nucleon current in addition to the usual V and A terms (P,S,T in SUSY contributions) contains momentum dependent terms [23].

Within the impulse approximation the nuclear current  $J_L^\rho$  in Eq. (1) expressed with nucleon fields  $\Psi$  takes the form

$$J_L^{\mu\dagger} = \bar{\Psi} \tau^+ \left[ g_V(q^2) \gamma^\mu - i g_M(q^2) \frac{\sigma^{\mu\nu}}{2m_p} q_\nu - g_A(q^2) \gamma^\mu \gamma_5 + g_P(q^2) q^\mu \gamma_5 \right] \Psi, \quad (22)$$

where  $M$  is the nucleon mass,  $q^\mu = (p - p')_\mu$  is the momentum transferred from hadrons to leptons ( $p$  and  $p'$  are four momenta of neutron and proton, respectively) and  $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ .  $g_V(q^2)$ ,  $g_M(q^2)$ ,  $g_A(q^2)$  and  $g_P(q^2)$  are real functions of a Lorenz scalar  $q^2$ . The values of these form factors in the zero-momentum transfer limit are known as the vector, weak-magnetism, axial vector and induced pseudoscalar coupling constants, respectively.

For nuclear structure calculations it is necessary to reduce the nucleon current to the non-relativistic form. We shall neglect small energy transfers between nucleons in the non-relativistic expansion. Then the form of the nucleon current coincides with those in the Breit frame and we arrive at [58],

$$J^\mu(\vec{x}) = \sum_{n=1}^A \tau_n^+ [g^{\mu 0} J^0(\vec{q}^2) + g^{\mu k} J_n^k(\vec{q}^2)] \delta(\vec{x} - \vec{r}_n), \quad k = 1, 2, 3, \quad (23)$$

with

$$J^0(\vec{q}^2) = g_V(q^2), \quad \vec{J}_n(\vec{q}^2) = g_M(\vec{q}^2) i \frac{\vec{\sigma}_n \times \vec{q}}{2M} + g_A(\vec{q}^2) [\vec{\sigma} - \frac{\vec{q} \vec{\sigma}_n \cdot \vec{q}}{q^2 + m_\pi^2}] \quad (24)$$

$\vec{r}_n$  is the coordinate of the  $n$ th nucleon.

For the form factors we shall use the following parameterization [23]:

$$g_V(\vec{q}^2) = g_V / (1 + \vec{q}^2 / \Lambda_V^2)^2, \quad g_M(\vec{q}^2) = (\mu_p - \mu_n) g_V(\vec{q}^2), \\ g_A(\vec{q}^2) = g_A / (1 + \vec{q}^2 / \Lambda_A^2)^2$$

where  $g_V = 1$ ,  $g_A = 1.254$ ,  $(\mu_p - \mu_n) = 3.70$ ,  $\Lambda_V^2 = 0.71 \text{ (GeV)}^2$  [59] and  $\Lambda_A = 1.09 \text{ GeV}$  [60]. In previous calculations only one general cut-off  $\Lambda_V = \Lambda_A \approx 0.85 \text{ GeV}$  was used. In this work we take the empirical value of  $\Lambda_A$  deduced from the antineutrino quasielastic reaction  $\bar{\nu}_\mu p \rightarrow \mu^+ n$ . A larger value of the cut-off  $\Lambda_A$  is expected to increase slightly the values of corresponding nuclear matrix elements. It worth noting that with these modifications of the nuclear current one gets a new contribution in the neutrino mass mechanism, namely the tensor contribution. The two body effective transition operator takes in momentum space the form

$$\Omega = \tau_+ \tau_+ (-g_V(\vec{q}^2) + h_{GT} \sigma_{12} - h_T S_{12}) \quad (25)$$

where the three terms correspond to Fermi (F), Gamow-Teller (GT) and Tensor (T). One finds that

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q}) - \sigma_{12}, \quad \sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2. \quad (26)$$

Note that the tensor operator is defined in momentum space ( $\hat{q}$  rather than  $\hat{r}$ ) and there is a change of sign in going to the coordinate space.

$$\frac{h_{GT}(\vec{q}^2)}{g_A^2(\vec{q}^2)} = [1 - \frac{2}{3} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} + \frac{1}{3} (\frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2})^2] + \frac{2}{3} \frac{g_M^2(\vec{q}^2) \vec{q}^2}{4m_p^2},$$

Table 1. The Fermi, Gamow-Teller and Tensor nuclear matrix elements for the light Majorana neutrino exchange of the  $0\nu\beta\beta$ -decay of  $^{76}\text{Ge}$  and  $^{130}\text{Te}$  with ( rows 2 and 4) and without (rows 1 and3) short-range correlations.

transition	Gamow-Teller			Tensor		$M_F^{light}$	$M_{GT}^{light}$	$M_T^{light}$
	AA	AP	PP	AP	PP			
$^{76}\text{Ge}$	5.132	-1.392	0.302	-0.243	0.054	-2.059	4.042	-0.188
	2.797	-0.790	0.176	-0.246	0.055	-1.261	2.183	-0.190
$^{130}\text{Te}$	4.158	-1.173	0.258	-0.329	0.074	-1.837	3.243	-0.255
	1.841	-0.578	0.134	-0.333	0.075	-1.033	1.397	-0.258

Table 2. Nuclear matrix elements for the light and heavy Majorana neutrino exchange modes of the  $0\nu\beta\beta$ -decay for the nuclei studied in this work calculated within the renormalized pn-QRPA.

M. E.	$(\beta\beta)_{0\nu} - \text{decay} : 0^+ \rightarrow 0^+$ transition								
	$^{76}\text{Ge}$	$^{82}\text{Se}$	$^{96}\text{Zr}$	$^{100}\text{Mo}$	$^{116}\text{Cd}$	$^{128}\text{Te}$	$^{130}\text{Te}$	$^{136}\text{Xe}$	$^{150}\text{Nd}$
light Majorana neutrino (I=light)									
$M_{VV}^I$	0.80	0.74	0.45	0.82	0.50	0.75	0.66	0.32	1.14
$M_{AA}^I$	2.80	2.66	1.54	3.30	2.08	2.21	1.84	0.70	3.37
$M_{PP}^I$	0.23	0.22	0.15	0.26	0.15	0.24	0.21	0.11	0.35
$M_{AP}^I$	-1.04	-0.98	-0.65	-1.17	-0.69	-1.04	-0.91	-0.48	-1.53
$M_{<m_\nu>}^I$	2.80	2.64	1.49	3.21	2.05	2.17	1.80	0.66	3.33
heavy Majorana neutrino (I= heavy)									
$M_{VV}^I$	23.9	22.0	16.1	28.3	17.2	25.8	23.4	13.9	39.4
$M_{MM}^I$	-55.4	-51.6	-38.1	-67.3	-39.8	-60.4	-54.5	-31.3	-92.0
$M_{AA}^I$	106.	98.3	68.4	123.	74.0	111.	100.	58.3	167.
$M_{PP}^I$	13.0	12.0	9.3	16.1	9.1	14.9	13.6	7.9	23.0
$M_{AP}^I$	-55.1	-50.7	-41.1	-70.1	-39.0	-64.9	-59.4	-34.8	-101.
$M_{\eta_N}^I$	32.6	30.0	14.7	29.7	21.5	26.6	23.1	14.1	35.6

$$\frac{h_T(\vec{q}^2)}{g_A^2(\vec{q}^2)} = \left[ \frac{2}{3} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} - \frac{1}{3} \left( \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2} \right)^2 \right] + \frac{1}{3} \frac{g_M^2(\vec{q}^2)\vec{q}^2}{4m_p^2}, \quad (27)$$

The exact results will depend on the details of the nuclear model, since the new operators have different momentum (radial) dependence than the traditional ones and the tensor component is entirely new. We can get a crude idea of what is happening by taking the above average momentum  $\langle q \rangle = 100$  MeV/c. Then we find that the GT ME is reduced by 22%. Then assuming that T matrix element is about half the GT one, we find that the total reduction is 28%. This is in perfect agreement with the exact results for the A=76 system, 29%, but a bit smaller than the 38% obtained for the A=130 system. We will now summarize the results obtained with

the above modifications of the nucleon current for light neutrino (see Eq. (5)) for the two representative  $0\nu\beta\beta$ -decay nuclei  $^{76}\text{Ge}$  and  $^{130}\text{Te}$  in Table 1. The details of our calculations will be given elsewhere [23]. One notices significant additional contributions to GT (AP and PP) and tensor (AA and PP) nuclear matrix elements coming from higher order nucleon current terms. AP and PP originate from the second (first) and third (second) terms in  $h_{GT}(h_T)$  of Eq. (27).

By glancing at the Table 1 we also see that, with proper treatment of short-range two-nucleon correlations (see e.g. Vergados [6]), all matrix elements are strongly suppressed. The effect is even stronger in the case of heavy intermediate particles. Detailed results [23] for various nuclei are presented in Table 2.

## 5. Extraction of the lepton violating parameters

The limits deduced for the lepton-number violating parameters depend on the values of nuclear matrix element, of the kinematical factor and of the current experimental limit for a given isotope [see Eq. (5)].

### 5.1. Traditional lepton violating parameters

Even, though, we expect the nuclear matrix elements entering the light neutrino mass mechanism to be decreased by about 30% , independently of the nuclear model, we will stick to the calculations as reported. Thus the present best experimental limits [61]–[71] can be converted to upper limits on  $\langle m_\nu \rangle$  and  $\eta_N$ .

The thus obtained results are given in Table 4. The references of Table 4 are defined as follows: Ex1=Heidelberg-Moscow Collaboration [61], Ex2=Elliott *et al* [62], Ex3=Kawashima *et al* [63], Ex4=Ejiri *et al* [64], Ex5=Davenich *et al* [65], Ex6=Bernatovicz *et al* [66], Ex7=Alessandrello *et al* [67], Ex8=De Silva *et al* [68], Ex9=Busto *et al* [69]. Thus, the most restrictive limits are as follows:

$$\langle m_\nu \rangle^{best} < 0.62 \text{ eV} \quad , \quad \langle \eta_N \rangle^{best} < 1.0 \times 10^{-7} \quad (28)$$

(see Ref.[23], [61]) By assuming  $\langle m_\nu \rangle = \langle m_\nu \rangle^{best}$  and  $\eta_N = \eta_N^{best}$ . (5) we calculated half-lives of the  $0\nu\beta\beta$ -decay  $T_{1/2}^{exp-0\nu}(\langle m_\nu \rangle^{best})$ ,  $T_{1/2}^{exp-0\nu}(\eta_N^{best})$  for nuclear systems of interest using specific mechanisms with the "best" parameters. The thus obtained results are given in Table 4. The references of Table 4 are defined as follows: Ex1=Heidelberg-Moscow Collaboration [61], Ex2=Elliott *et al* [62], Ex3=Kawashima *et al* [63], Ex4=Ejiri *et al* [64], Ex5=Davenich *et al* [65], Ex6=Bernatovicz *et al* [66], Ex7=Alessandrello *et al* [67], Ex8=Busto *et al* [69]. Ex9=De Silva *et al*

Table 3. The present state of the Majorana neutrino mass searches in  $\beta\beta$ -decay experiments.  $T_{1/2}^{exp-0\nu}$ (present) is the best presently available lower limit on the half-life of the  $0\nu\beta\beta$ -decay for a given isotope. The corresponding upper limits on lepton number non-conserving parameters  $< m_\nu >$  and  $\eta_N$  are presented. For the definition of the references and "best" see text.

Nucleus	$^{76}Ge$	$^{82}Se$	$^{96}Zr$	$^{100}Mo$	$^{116}Cd$
$T_{1/2}^{exp-0\nu}$ (present) [y]	$1.1 \times 10^{25}$	$2.7 \times 10^{22}$	$3.9 \times 10^{19}$	$5.2 \times 10^{22}$	$2.9 \times 10^{22}$
Ref.	[Ex1]	[Ex2]	[Ex3]	[Ex4]	[Ex5]
$< m_\nu >$ [eV]	0.62	6.3	203.	2.9	5.9
$T_{1/2}^{exp-0\nu}$ [y]					
( $< m_\nu >^{best}$ )	$1.1 \times 10^{25}$	$2.8 \times 10^{24}$	$4.2 \times 10^{24}$	$1.2 \times 10^{24}$	$2.6 \times 10^{24}$
$\eta_N$	$1.0 \times 10^{-7}$	$1.1 \times 10^{-6}$	$4.0 \times 10^{-5}$	$6.2 \times 10^{-7}$	$1.1 \times 10^{-6}$
$T_{1/2}^{exp-0\nu}(\eta_N^{best})$ [y]	$1.1 \times 10^{25}$	$2.9 \times 10^{24}$	$5.8 \times 10^{24}$	$1.8 \times 10^{24}$	$3.2 \times 10^{24}$

Nucleus	$^{128}Te$	$^{130}Te$	$^{136}Xe$	$^{150}Nd$
$T_{1/2}^{exp-0\nu}$ (present) [y]	$7.7 \times 10^{24}$	$8.2 \times 10^{21}$	$4.2 \times 10^{23}$	$1.2 \times 10^{21}$
Ref.	[Ex6]	[Ex7]	[Ex8]	[Ex9]
$< m_\nu >$ [eV]	1.8	13.	4.9	8.5
$T_{1/2}^{exp-0\nu}$				
( $< m_\nu >^{best}$ ) [y]	$6.6 \times 10^{25}$	$3.8 \times 10^{24}$	$2.7 \times 10^{25}$	$2.3 \times 10^{23}$
$\eta_N$	$2.9 \times 10^{-7}$	$2.0 \times 10^{-6}$	$4.5 \times 10^{-7}$	$1.6 \times 10^{-6}$
$T_{1/2}^{exp-0\nu}(\eta_N^{best})$ [y]	$5.9 \times 10^{25}$	$3.1 \times 10^{24}$	$7.9 \times 10^{24}$	$2.7 \times 10^{23}$

al [68], Since the quantities  $< m_\nu >$ ,  $\eta_N$  depend only on particle theory parameters these quantities indicate the experimental half-life limit for a given isotope, which the relevant experiments should reach in order to extract the best present bound on the corresponding lepton number violating parameter from their data. Some of them have a long way to go to reach the  $Ge$  target limit.

A summary involving most of the available nuclear matrix elements and taking into account what, at present, is a good guess as canonical values of the lepton violating parameters is provided in Tables 4 and 5. The references in these tables are defined as follows: R=Retamosa *et al* [36], H=Haxton *et al* [30], E1=Engel *et al* [45], E2=Engel *et al* [42], S=Suhonen *et al* [28], M=Muto *et al* [44], T=Tomoda *et al* [7], P1=Pantis *et al* [12], P2=Pantis *et al* [12] (p-n pairing), S1=Simkovic *et al* [51] (and private communication), F=Faessler *et al* [18], [19] P=Present calculation (see Simkovic *et al* [23] for the nuclear Matrix elements). Notice in particular that the present calculation, marked P in the table, involves not only renormalized QRPA [24, 54], but takes into account the corrections in the hadronic current [23] discussed above (see table 4).

Table 4. The lifetimes predicted for  $0^+ \rightarrow 0^+ 0\nu\beta\beta$ -decay in various mechanisms (light neutrino, heavy neutrino,  $\lambda$  and  $\eta$  terms and SUSY contribution) for suitable input of lepton violating parameters and available nuclear calculations. For the definitions of the references see text.

Ref.	$(\beta\beta)_{0\nu} - \text{decay} : 0^+ \rightarrow 0^+ \text{ transition}$ $T_{1/2}^{0\nu-theor}(\langle m_\nu \rangle, \langle \lambda \rangle, \langle \eta \rangle, \langle \eta_N \rangle, \langle \eta_{SUSY} \rangle)$									
	$^{48}\text{Ca}$	$^{76}\text{Ge}$	$^{82}\text{Se}$	$^{96}\text{Zr}$	$^{100}\text{Mo}$	$^{116}\text{Cd}$	$^{128}\text{Te}$	$^{130}\text{Te}$	$^{136}\text{Xe}$	$^{150}\text{Nd}$
	$10^{24}$	$10^{24}$	$10^{24}$	$10^{24}$	$10^{24}$	$10^{24}$	$10^{25}$	$10^{24}$	$10^{24}$	$10^{22}$
	$\langle m_\nu \rangle = 1\text{eV}, \langle \lambda \rangle = 0, \langle \eta \rangle = 0, \langle \eta_N \rangle = 0, \langle \eta_{SUSY} \rangle = 0$									
R	12.8	34.8	4.80						24.2	
H	6.34	3.36	1.16				0.80	0.32		
E1		4.60	1.84				0.90	0.48		
E2		28.0	11.2				3.00	1.32	6.60	
S		8.12	2.86				3.60	1.66		
M		4.66	1.20		2.54		1.54	0.98	4.42	6.74
T		4.32	1.22		0.52		1.96	1.08	2.80	8.90
P1	5.00	7.20	3.00	1.22	7.80	9.40	3.80	1.72	6.60	
P2	56.0	36.0	5.60	54.0		9.80	30.0	4.20	5.60	
S1		17.9			0.50	1.44	2.18		17.5	
P		4.22	1.08	1.61	0.46	0.99	2.53	1.46	10.1	8.78
	$\langle m_\nu \rangle = 0, \langle \lambda \rangle = 10^{-6}, \langle \eta \rangle = 0, \langle \eta_N \rangle = 0, \langle \eta_{SUSY} \rangle = 0$									
R	7.45	50.2	3.25						22.2	
S		7.75	1.14				14.8	0.89		
M		7.35	0.99		0.95		13.5	0.95	4.90	3.73
T		8.02	1.07		0.55		21.1	1.18	3.47	6.71
P1	2.71	8.90	2.08	0.94	30.6	39.1	22.7	1.34	2.73	
P2	27.9	41.2	4.39	27.7	10.3	10.8	165	2.22	4.42	

### 5.2. R-parity induced lepton violating parameters

In this section we will elaborate a bit further on the R-parity violating parameters. We will consider the pionic contribution (1). We will first attempt to evaluate the relevant amplitude using harmonic oscillator wave functions, but adjusting the parameters to fit related experiments.

Let us begin with the second process of Eq. (1). This process involves a direct term and an exchange term. The direct term is nothing but a decay of the pion into two leptons with a simultaneous change of a neutron to a proton by the relevant nucleon current, which in this case can only be of the of the PS type. The tensor contribution cannot lead to a pseudoscalar coupling at the nucleon level, which is needed to be coupled to the usual pion nucleon coupling in the other vertex to get the relevant operator for a  $0^+ \rightarrow 0^+$  decay. Thus the amplitude involving the meson is related to  $\pi, \mu$  decay.

$$A_{1\pi}(\text{direct}) = 4 \tilde{\alpha}_{1\pi} \frac{\sigma_1 \cdot \vec{q}}{(2 m_N)} m_\pi^2 \exp\left(-\frac{(qb)^2}{6}\right) \quad (29)$$

Table 5. The previous table continued.

	$\langle m_\nu \rangle = 0, \langle \lambda \rangle = 0, \langle \eta \rangle = 10^{-8}, \langle \eta_N \rangle = 0, \langle \eta_{SU_{SY}} \rangle = 0$									
R	6.42	27.2	6.24						22.2	
S		36.7	11.1				10.7	5.92		
M		7.35	0.99		0.95		13.5	0.95	4.90	3.73
T		2.25	0.65		0.28		0.67	0.44	1.21	3.39
P1	15.11	3.10	6.51	1.48	3.44	19.2	1.20	0.62	1.23	
P2	43.2	22.8	5.16	7.95	102	83.2	1.90	1.05	0.96	
	$\langle m_\nu \rangle = 0, \langle \lambda \rangle = 0, \langle \eta \rangle = 0, \langle \eta_N \rangle = 10^{-7}, \langle \eta_{SU_{SY}} \rangle = 0$									
P1	4.95	0.25	3.35	67.1	4.70	23.5	0.78	3.03	1.42	
P2	124	0.59	7.23	671	1.47	33.6	1.27	1.31	1.01	
P		15.4	4.10	8.10	0.97	8.40	8.51	4.54	3.94	40.6
	$\langle m_\nu \rangle = 0, \langle \lambda \rangle = 0, \langle \eta \rangle = 0, \langle \eta_N \rangle = 0, \langle \eta_{SU_{SY}} \rangle = 10^{-8}$									
F		3.3	0.86	0.71	0.30	0.85	0.93	0.45	1.2	3.2
P		4.5	1.0	1.4	0.59	1.4	1.5	0.71	2.0	5.2

with

$$\tilde{\alpha}_{1\pi} = (2\pi)^3 \frac{m_N}{3 m_q} \chi(0) \quad (30)$$

The exchange contribution, in which the produced up quark of the meson is not produced from the "vacuum" but it comes from the initial nucleon, is a bit more complicated. The harmonic oscillator quark model, however can be used to get its relative magnitude (including the sign) with respect to the direct term. This way we find

$$A_{1\pi}(\text{exchange}) = -3 \tilde{\alpha}_{1\pi} \left[ \frac{\sigma_1 \cdot \vec{q}}{(2 m_N)} m_\pi^2 \exp\left(-\frac{(qb)^2}{6}\right) \right] \quad (31)$$

Thus the effective two-body transition operator in momentum space at the nucleon level becomes:

$$\Omega_{1\pi}^{PS} = c_{1\pi} \left[ \frac{\sigma_1 \cdot \vec{q} \sigma_2 \cdot \vec{q}}{(2 m_N)^2} \exp\left(-\frac{(qb)^2}{6}\right) \right] \frac{m_\pi^2}{q^2 + m_\pi^2} \quad (32)$$

with  $c_{1\pi} = g_r \tilde{\alpha}_{1\pi}$  i.e.

$$c_{1\pi} = (2\pi)^3 \frac{m_N}{3 m_q} g_r \chi(0) \quad (33)$$

where  $g_r = 13.5$  is the pion nucleon coupling and  $m_q$  is the constituent quark mass. Note the presence of the exponential form factor in the harmonic oscillator model, which has been ignored in other treatments. We see that, in going from the quark to the nucleon level, the factor of three coming from the mass gain is lost due to the momentum being reduced by a factor of three. The quantity  $\chi(0)$  is essentially the meson wave function at the origin given by:

$$\chi(0) = \frac{\sqrt{6}}{2^{1/4}} m_\pi^{-3/2} \psi(0) \quad (34)$$



The quantity  $\chi(0)$  can be obtained from the  $\pi \rightarrow \mu, \nu$  decay via the expression

$$\frac{1}{\tau} = \frac{1}{\pi} \left( \frac{G_F}{\sqrt{2}} \right)^2 m_\pi^2 m_\mu^2 \left( 1 - \frac{m_\pi^2}{m_\mu^2} \right)^2 \chi^2(0) \quad (35)$$

From the measured lifetime  $\tau = 2.6 \times 10^{-8}$  we obtain  $\chi(0) = 0.46$

The first process of Eq. 1 is easier to handle. Now both the PS and T terms contribute. We thus get

$$A_{2\pi}(T) = \frac{3}{8} \tilde{\alpha}_{2\pi} m^4, A_{2\pi}(PS) = \frac{1}{8} \tilde{\alpha}_{2\pi} m^4, \quad (36)$$

$$\tilde{\alpha}_{2\pi} = 4 (2\pi)^3 \chi^2(0) \quad (37)$$

$$\Omega_{2\pi}^T = c_{2\pi} \frac{\sigma_1 \cdot \vec{q} \sigma_2 \cdot \vec{q}}{(2 m_N)^2} \frac{m_\pi^4}{(q^2 + m_\pi^2)^2}, \quad \Omega_{2\pi}^{PS} = \frac{2}{3} \Omega_{2\pi}^T \quad (38)$$

$$\tilde{\alpha}_{2\pi} = 4 (2\pi)^3 \chi^2(0), \quad c_{2\pi} = 4 (2\pi)^3 g_r^2 \chi^2(0) \quad (39)$$

Using the above value of  $\chi(0)$  and  $g_r = 13.5$  we get  $c_{1\pi} = 109$  and  $c_{2\pi} = 198$  which are in good agreement with the values 132.4 and 170.3 respectively obtained by Faessler et al [18].

It is now customary, but it can be avoided [25], to go to coordinate space and express the nuclear matrix elements in the same scale with the standard matrix elements involving only nucleons. Thus we get

$$ME_k = \left( \frac{m_A}{m_p} \right)^2 \alpha_{k\pi} \frac{m_p}{m_e} [M_{GT}^{k\pi} + M_T^{k\pi}] \quad (40)$$

Where the two above matrix elements are the usual GT and T matrix elements with the additional radial dependence given by

$$F_{GT}^{1\pi} = e^{-x}, \quad F_T^{1\pi} = (3 + 3x + x^2) \frac{e^{-x}}{x} \quad (41)$$

$$F_{GT}^{2\pi} = (x - 2)e^{-x}, \quad F_T^{2\pi} = (1 + x) e^{-x} \quad (42)$$

$$\alpha_{1\pi} = -c_{1\pi} \rho, \alpha_{2\pi} = c_{2\pi} \rho, \quad \rho = \frac{1}{48 f_A^2} \left( \frac{m_\pi}{m_p} \right)^4 \left( \frac{m_p}{m_A} \right)^2 \quad (43)$$

In the above formulas we have tried to stick to the definition of  $\eta_{SUSY}$  given above (see 11), but since the tensor (at the quark level) does not contribute to the  $1\pi$  diagram the "effective" nuclear matrix element is not the sum of the two matrix elements of Eq. (40), but only  $ME_2$ , and  $\chi_{PS}$  depends on the nuclear matrix elements, i.e.

$$ME_{eff} = ME_2, \quad \chi_{PS} = \frac{2}{3} \left( 4 \frac{ME_1}{ME_2} + 1 \right) \quad (44)$$

There is no difference, of course, between the two expressions if  $ME_2$  is dominant, as is actually the case).

Before proceeding further we should remark that for the experimentally derived harmonic oscillator parameter for the  $\pi$  meson,  $b = 1.8 f$ ,  $\alpha_{2\pi}$  is dominant and  $\chi_{PS}$  approaches the value of  $2/3$ . In fact we find  $\alpha_{1\pi} = -1.2 \times 10^{-2}$  and  $\tilde{\alpha}_{1\pi} = 0.15$  which are in good agreement with the values  $-4.4 \times 10^{-2}$  and  $0.20$  respectively obtained by Faessler et al [18]. Furthermore from the nuclear matrix elements of Ref. [18] one can see that the  $M^{2\pi}$  is favored, since, among other things, its tensor and Gamow-Teller components are the same magnitude and sign (in the  $1\pi$  mode they are opposite). Thus nuclear physics also favors the  $2\pi$  mode.

With the above ingredients and using the nuclear matrix elements of [18] we can extract from the data values of  $\eta_{SUSY}$ .

Then one can use these values of  $\eta_{SUSY}$  in order to extract values for the R-parity violating parameters  $\lambda'_{111}$ .

As we have already mentioned one must start with 5 parameters in the allowed SUSY parameter space and solve the RGE equations to obtain the values of the needed parameters at low energies [72]–[74]. For our purposes is adequate to utilize typical parameters, which have already appeared in the literature [72], [73]. One then finds

$$\lambda'_{111} = C_{\tilde{\chi}^0}(\eta_{SUSY})^{1/2} \quad (\text{neutralinos only}) \quad (45)$$

$$\lambda'_{111} = C_{\tilde{g}}(\eta_{SUSY})^{1/2} \quad (\text{gluino only}) \quad (46)$$

When both neutralinos and gluinos are included we write

$$\lambda'_{111} = C_{\tilde{\chi}^0, \tilde{g}}(\eta_{SUSY})^{1/2} \quad (47)$$

The values of these coefficients are given in Tab. 6 for the nine SUSY models mentioned above. From Table 6 we see that there is quite spread in the quantities  $C_{\tilde{\chi}^0}$ ,  $C_{\tilde{g}}$  and  $C_{\tilde{\chi}^0, \tilde{g}}$ , depending on the SUSY parameter space. We will see that this is the largest uncertainty in estimating the SUSY contribution to  $0\nu\beta\beta$  decay. In all of these cases the intermediate selectron-neutralino mechanism appears to be the most dominant. The most favorable situation occurs in the case # 7 of Table 6. And this what we will consider in extracting the limits on  $\lambda'_{111}$ . Combining the above values of the couplings  $\alpha_{k\pi}$ ,  $k=1,2$ , with the corresponding nuclear matrix elements of Faessler et al [18] (F) and the two nucleon ME of Wodecki et al [74] (W) we obtain the limits listed as Pr in Table 7.

Thus the most stringent limit is obtained from the  $^{76}\text{Ge}$  data and is

$$\lambda'_{111} \leq 6.0 \times 10^{-4} \quad (\text{for case \#7}) \quad (48)$$

The above quantities are assumed positive. If not, the absolute value is understood.

Table 6. A sample of relevant parameters obtained by some choices in the allowed SUSY parameter space. It clear that in all cases the neutralino mediated mechanism is dominant (for definitions see text). The parameters C shown have been multiplied by  $10^{-3}$

input	Kane et al (# 1-3)			Ramond et al (# 4-9)					
	# 1	#2	#3	#4	#5	#6	#7	#8	#9
$\tan\beta$	10.	1.5	5.0	5.4	2.7	2.7	5.2	2.6	6.3
$m_{\chi_1^0}$	124.	26	96	83	124	58	34	34	50
$m_{\chi_2^0}$	237.	65	173	150	204	108	66	74	92
$m_{\chi_3^0}$	455.	219	310	391	445	336	170	191	208
$m_{\chi_4^0}$	471.	263	342	409	472	361	208	236	244
$m_{\tilde{e}_L}$	328.	124	211	426	472	310	90	94	109
$m_{\tilde{u}_L}$	700.	283	570	590	664	449	251	275	319
$m_{\tilde{d}_R}$	676.	276	550	577	638	441	246	268	310
$m_{\tilde{g}}$	718.	292	610	483	706	371	280	304	350
$C_{\tilde{\chi}^0} \times 10^{-3}$	3.3	0.023	0.46	5.9	14	1.4	0.0068	0.0089	0.019
$C_{\tilde{g}} \times 10^{-3}$	14	1.6	54	56	110	13	0.97	1.5	3.1
$C_{\tilde{\chi}^0, \tilde{g}} \times 10^{-3}$	3.2	0.023	0.45	5.3	12	1.3	0.0068	0.0089	0.019

Table 7. The limits for  $\eta_{SUSY}$  and  $\lambda'_{111}$  obtained : a) For the pion mechanism using the values of  $\alpha_{1\pi}$  and  $\alpha_{2\pi}$  computed in this work and the nuclear ME of Faessler et al (F) b) Using the nuclear ME of the two nucleon mode of Ref. Wodecki et al (W). In extracting the values of  $\lambda'_{111}$  we used the SUSY data of #7 of Table 6. The experimental lifetimes employed are those of Table 4.

(A,Z)	Pion mode				Only nucleons	
	$\eta_{SUSY}(Pr)$	$\lambda'_{111}(Pr)$	$\eta_{SUSY}(F)$	$\lambda'_{111}(F)$	$\eta_{SUSY}(Pr)$	$\lambda'_{111}(Pr)$
$^{76}Ge$	$8.4 \times 10^{-9}$	$6.0 \times 10^{-4}$	$5.5 \times 10^{-9}$	$4.8 \times 10^{-4}$	$2.6 \times 10^{-8}$	$1.1 \times 10^{-3}$
$^{100}Mo$	$3.2 \times 10^{-8}$	$1.8 \times 10^{-3}$	$2.4 \times 10^{-8}$	$1.1 \times 10^{-3}$	$1.1 \times 10^{-7}$	$2.2 \times 10^{-3}$
$^{116}Cd$	$7.6 \times 10^{-8}$	$1.8 \times 10^{-3}$	$5.4 \times 10^{-8}$	$1.5 \times 10^{-3}$	$2.6 \times 10^{-7}$	$3.3 \times 10^{-3}$
$^{128}Te$	$1.6 \times 10^{-8}$	$8.1 \times 10^{-4}$	$1.1 \times 10^{-8}$	$6.8 \times 10^{-4}$	$5.6 \times 10^{-8}$	$1.6 \times 10^{-3}$
$^{130}Te$	$1.0 \times 10^{-7}$	$1.8 \times 10^{-3}$	$5.5 \times 10^{-8}$	$1.5 \times 10^{-3}$	$1.3 \times 10^{-7}$	$6.5 \times 10^{-3}$
$^{136}Xe$	$2.4 \times 10^{-8}$	$9.4 \times 10^{-4}$	$1.7 \times 10^{-8}$	$7.8 \times 10^{-4}$	$8.7 \times 10^{-7}$	$2.2 \times 10^{-3}$
$^{150}Nd$	$7.3 \times 10^{-8}$	$2.3 \times 10^{-3}$	$5.2 \times 10^{-8}$	$1.4 \times 10^{-3}$	$2.4 \times 10^{-7}$	$3.0 \times 10^{-3}$

## 6. Conclusions

We have seen that  $0\nu\beta\beta$  decay pops up in almost any fashionable particle model. Thus it can set useful limits not only on the light neutrino mass (28), but in addition on other lepton violating parameters like  $\langle \eta_N \rangle$  of (28) or the parameters  $\lambda$  and  $\eta$  (see sect. 2.2). Finally we mention again the limit extracted on the R-parity violating parameter (48). A set of limits, for our choice of nuclear matrix elements, derived from the various

Table 8. Summary of the results presented in this work.

	$\langle m_\nu \rangle$	$\langle \lambda \rangle$	$\langle \eta \rangle$	$\langle \eta_N \rangle$	$\langle \eta_{SUSY} \rangle$	$\lambda'_{111}$
(A,Z)	eV	$10 \times 10^{-6}$	$10 \times 10^{-8}$	$10 \times 10^{-8}$	$10 \times 10^{-8}$	$10 \times 10^{-4}$
	Pr	P1	P1	Pr	Pr	Pr
$^{76}Ge$	0.27	0.56	0.32	0.44	0.31	4.0
$^{100}Mo$	2.9	26	8.8	6.2	3.2	18
$^{116}Cd$	5.9	37	26	11	7.6	18
$^{128}Te$	1.8	5.6	1.3	2.9	1.6	8.1
$^{130}Te$	13	7.6	5.2	20	10	18
$^{136}Xe$	49	2.1	1.4	4.5	2.4	9.4
$^{150}Nd$	8.5	5.6	5.3	16	7.3	23

nuclear targets is given in Table 8. For  $^{76}Ge$  our results are different from the ones given above, since we have used here the unpublished new limit of the Heidelberg- Moscow experiment  $T_{1/2} \geq 5.7 \times 10^{25}$  yr.

We see that limits are quite stringent, but they, of course, have uncertainties in them. They come from nuclear physics, especially for the short ranged operators or from particle physics, as, e.g., in the case of supersymmetry. It is however evident that in the extraction of  $\lambda'_{111}$  the main uncertainty comes from the parameters of supersymmetry. After all  $\lambda'_{111}$  depends on the inverse fourth root of the lifetime and the inverse square root of the nuclear matrix elements. On the other hand it depends in the second power of the masses of the mediating SUSY scalars. This is not true of the models considered here but it is found in other calculations as well [74].

It is clear that during the last year the interest of most people is being focused on the light neutrino mass mechanism. This due to the experimental indications for neutrino oscillations of solar (Homestake [75], Kamiokande [76], Gallex [77] and SAGE [78]), atmospheric (Kamiokande [79], IMB [80] and Soudan [81], Super-Kamiokande [82]) and terrestrial (LSND experiment [83]) experiments.

One can use the constraints imposed by the results of neutrino oscillation experiments on  $\langle \Delta m_\nu^2 \rangle$ . These experiments, of course, cannot predict the scale of the masses or the Majorana phases. The predictions differ from each other due the different input and structure of the neutrino mixing matrix and assumptions. Bilenky et al [84] and others [86] have shown that under quite reasonable assumptions in a general scheme with three light Majorana neutrinos and mass hierarchy  $| \langle m_\nu \rangle |$  is smaller than  $10^{-2}$  eV. In another study outlined in Ref.[85] the authors end up with  $| \langle m_\nu \rangle | \approx 0.14$  eV. Thus one can see that, the current limit on  $\langle m_\nu \rangle$  in (28) is quite a bit higher than the neutrino oscillation data.

There is a new experimental proposal for measurement of the  $0\nu\beta\beta$ -

decay of  $^{76}\text{Ge}$ , which intends to use 1 ton (in an extended version 10 tons) of enriched  $^{76}\text{Ge}$  and to reach the half-life limit  $T_{1/2}^{0\nu\text{-exp}} \geq 5.8 \times 10^{27}$  and  $T_{1/2}^{0\nu\text{-exp}} \geq 6.4 \times 10^{28}$  after one and 10 years of measurements, respectively. From these half-life values one can deduce [see Eq. (5) and Table 2] the possible future limits on the effective light neutrino mass  $2.7 \times 10^{-2}$  eV and  $8.1 \times 10^{-3}$  eV, respectively. From the comparison with limits advocated by the neutrino oscillation phenomenology we conclude that GENIUS experiment [61, 87] would be able to measure this lepton number violating process, provided, of course, that the neutrinos are Majorana particles.

We must emphasize that the plethora of other  $0\nu\beta\beta$ -decay mechanisms predicted by GUT's and SUSY do not diminish the importance of this reaction in settling the outstanding neutrino properties. One can show that the presence of these exotic mechanisms implies that the neutrinos are massive Majorana particles, even if the mass mechanism is not the dominant one [10, 88].

Thus one can say with certainty that the experimental detection of the  $0\nu\beta\beta$ -decay process would be a major achievement with important implications on the field of particle and nuclear physics as well as on cosmology.

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